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Reference satellite selection method for GNSS high-precision relative positioning

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ABSTRACT

Selecting the optimal reference satellite is an important component of high-precision relative positioning because the reference satellite directly influences the strength of the normal equation. The reference satellite selection methods based on elevation and positional dilution of precision (PDOP) value were compared. Results show that all the above methods cannot select the optimal reference satellite. We introduce condition number of the design matrix in the reference satellite selection method to improve structure of the normal equation, because condition number can indicate the ill condition of the normal equation. The experimental results show that the new method can improve positioning accuracy and reliability in precise relative positioning.

1. Introduction

Selecting the appropriate reference satellite is important in high-precision relative positioning because the reference satellite not only generates errors in all double-difference (DD) observations but also directly influences the structural stability of the functional model in the least squares solution. The choice of reference satellite will not only affect the structural stability of the normal equation matrix if observations are rejected, but also a change of reference satellite is equivalent to an ambiguity Z-transformation which preserves the underlying vector space of the normal equations.

The satellite with the highest elevation is generally selected as the reference satellite in DD equations because its observation has the lowest noise in most cases [1]. However, the observations of the satellite with the highest elevation may have large errors or even outliers in the obstacle environment because of the signal diffraction and multipath effects. Furthermore, the structural stability of the normal equation matrix in least squares estimation (LSE) is not considered in this method with the highest elevation [2].

The dilution of precision (DOP) value and condition number of the normal equation matrix are used to evaluate the structural stability of the normal equation matrix in the existing reference satellite selection strategies [3]. However, the DOP value is in fact the strength indicia of the satellite–receiver geometry in point positioning mode. As is known, in relative positioning mode, each DD observation equation includes the reference satellite observation. Thus, the DOP value is not good to evaluate the strength of the geometry or the structural stability of the normal equation matrix [4,5].

As the condition number of the matrix is a good indicator of the structural stability [6], it is proposed to be considered in the reference satellite selection. In this study, we proposed a new method to select the reference satellite by condition number of the functional model which can indicate the structural stability of the normal equation matrix. Real combined Global Positioning System/Global Navigation Satellite System/BeiDou Navigation Satellite System (GPS/GLONASS/BDS) data collected in the field are used to test the effectiveness of the new method.

2. Fundamental mathematical model of precise relative positioning

The functional and stochastic models are requisite components of the observation equations in GNSS data processing procedure. Firstly, this section deals with the functional model. If we neglect

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the phase deviation and hardware delay in the observation data, the original observation equation can be expressed as follows [7]:

$$\begin{cases} L_i^p = \rho_i^p + \mathrm{c}\cdot(dts^p - dtr_i) + \lambda^p\cdot N_i^p + I_i^p + T_i^p + \varepsilon_i^p \\ \varepsilon_i^p \sim N\left(0, \left(\sigma_i^p\right)^2\right) \end{cases} \tag{1}$$

where the superscript p represents the available satellite; subscript i represents the user receiver; L is the carrier phase observation (unit: m); ρ is the Euclidean distance from the satellite to the receiver antenna; dts and dtr denote the satellite clock and receiver clock, respectively; λ is the wavelength of the signal carrier; c is the speed of light in vacuum; N is the integer ambiguity of the carrier phase; I and T are the ionospheric delay and tropospheric delay, respectively.

Given that common errors of GNSS observations are significantly reduced in differential observation equations, especially in zero and short baseline processing, the DD carrier phase equation can be expressed as follows:

$$\begin{cases} L_{ij}^{pq} = \rho_{ij}^{pq} + \left(\lambda^p - \lambda^q\right)\cdot N_{ij}^q + \lambda^p\cdot N_{ij}^{pq} + \varepsilon_{ij}^{pq} \\ \varepsilon_{ij}^{pq} \sim N\left(0, \left(\sigma_{ij}^{pq}\right)^2\right) \end{cases} \tag{2}$$

where subscripts i and j represent the base and rover receivers, respectively; superscripts p and q are the visible satellite pairs, with p used as the reference satellite. If the satellite system adopts the Frequency Division Multiple Access (FDMA) coding technique, such as GLONASS, the carrier wavelength significantly differs from one satellite to another. Therefore, the DD equation of GLONASS is expressed in equation (2). By contrast, the DD equation of the satellite system, which utilizes the Code Division Multiple Access (CDMA) coding technique, e.g., GPS, Galileo, and BDS, can be derived as follows:

$$\begin{cases} L_{ij}^{pq} = \rho_{ij}^{pq} + \lambda^p\cdot N_{ij}^{pq} + \varepsilon_{ij}^{pq} \\ \varepsilon_{ij}^{pq} \sim N\left(0, \left(\sigma_{ij}^{pq}\right)^2\right) \end{cases} \tag{3}$$

From equations (2) and (3), with respect to the short baseline, the common errors, e.g., ionospheric delay and tropospheric delay have been significantly reduced. The remaining parameters in the observation equation mainly consists baseline components and integer ambiguities of the available satellites.

3. Ill condition of the normal equation

Equations (2) and (3) can be simplified as follows:

$$\boldsymbol{V} = \boldsymbol{HX} - \boldsymbol{L}, \boldsymbol{P} \tag{4}$$

where $\boldsymbol{V}$ is the residual matrix of the normal equation; $\boldsymbol{H}$ is the design matrix of the observables; $\boldsymbol{X}$ is the unknown variable matrix; $\boldsymbol{L}$ and $\boldsymbol{P}$ are the residual matrix and weight matrix of observables, respectively. Thus, the unknown parameters can be estimated using LSE as follows:

$$\begin{cases} \delta\boldsymbol{x} = \boldsymbol{N}^{-1}\boldsymbol{W} = \left(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{PH}\right)^{-1}\left(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{PL}\right) \\ \widehat{\boldsymbol{X}} = \boldsymbol{X}^0 + \delta\boldsymbol{x} \end{cases} \tag{5}$$

Given that the Euclidean distance between the satellite and receiver is greater than 20000 km, the errors in the design matrix has a negligible effect on the final solutions. If the errors in the design matrix are omitted and the errors in the residual matrix are assumed to be $\delta\boldsymbol{L}$, then the influence on $\delta\boldsymbol{x}$ can be determined using error propagating theory as follows [8]:

$$\begin{cases} \|\delta\boldsymbol{x}\| \le \left\|\boldsymbol{N}^{-1}\right\|\cdot\|\delta\boldsymbol{L}\| \\ \dfrac{\|\delta\boldsymbol{x}\|}{\|\boldsymbol{X}\|} \le \left\|\boldsymbol{N}^{-1}\right\|\cdot\|\boldsymbol{N}\|\cdot\dfrac{\|\delta\boldsymbol{L}\|}{\|\boldsymbol{L}\|} \end{cases} \tag{6}$$

where $\boldsymbol{N} = \boldsymbol{H}^{\mathrm{T}}\boldsymbol{PH}$, and $\|\cdot\|$ is the Euclidean norm of matrix. From equation (6), the absolute measurement error can be magnified $\left\|\boldsymbol{N}^{-1}\right\|$ times in the final solution, and the relative measurement error can be magnified $\left\|\boldsymbol{N}^{-1}\right\|\cdot\|\boldsymbol{N}\|$ times. As $\boldsymbol{N}$ is positive symmetric matrix, the matrix norm of the $\boldsymbol{N}$ is equal to the matrix norm of the $\boldsymbol{N}^{-1}$, i.e., $\left\|\boldsymbol{N}^{-1}\right\| = \|\boldsymbol{N}\|$. Therefore, in this study, $\left\|\boldsymbol{N}^{-1}\right\|$ is used to evaluate the ill condition of the normal equation.

4. GNSS reference satellite selection method

To reduce such common errors as ionospheric and tropospheric errors in the observables, the DD observation equations are extensively used in precise relative positioning. Thus, the reference satellite should be selected to derive differential equations between available satellite pairs. Given that the reference satellite serves an essential function in GNSS positioning, this section analyzes the reference satellite selection methods in data processing.

The common method of selecting the reference satellite is based on satellite elevation, which can be a good indicator to reflect the observable quality [9]. To be more precise, the signal path will become shorter when satellite elevation becomes higher, which mitigates the effects of other error sources, e.g., atmospheric delay and multipath, on the observations. Theoretically, the quality of the observation with the highest satellite elevation is the optimal. However, observation structure is related not only to satellite elevation but also to visible satellite distribution. Thus this method cannot ensure that the optimal observation structure, which directly affects the measurement errors of the final solutions, can be obtained.

Given that the observation structure serves a vital function in precise GNSS relative positioning, the positional dilution of precision (PDOP) value is adopted in selecting the reference satellite. PDOP can reflect the distribution of available satellites. The calculation formula of the PDOP value can be expressed as follows [10]:

$$PDOP = \sqrt{(\mathbf{Q})_{11} + (\mathbf{Q})_{22} + (\mathbf{Q})_{33}} \tag{7}$$

where $\boldsymbol{Q} = \left(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}\right)^{-1}$, and $\boldsymbol{H}$ is the design matrix of the DD equation. Although the PDOP value is considered an accuracy evaluation criterion in non-differential positioning, this value becomes inoperative in relative positioning. The fundamental cause of this phenomenon lies in the fact that the observation structure of the DD equation is not only related to the independent visible satellite distribution but is also involved in the relative distributions between reference and non-reference satellites [11]. In view of the aforementioned fact, the condition number, which is proven to be an effective evaluation of the ill condition level of the observation structure, is considered to select the reference satellite. The evaluation formula of the condition number is expressed in equation (6).

5. Data processing and results analysis

Two Trimble Net R9 receivers and Trimble Zephyr antennas were used to collect real data on the 10th May 2014 with a sampling interval of 30 s. The data in our experiment are collected in the static mode, but we processed the observables using the single epoch kinematic method. Given that the baseline is only 9.24 m, the DD ionospheric and tropospheric errors are assumed to be completely eliminated. Thus, the unknown variables only contain three baseline components and DD phase integer ambiguities for

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each satellite pairs. First, the float values of the ambiguities and their covariance matrix were estimated. Second, these ambiguities were fixed using the least-squares ambiguity decorrelation adjustment method (LAMBDA). The data were then collected under the mask environment, which can generate larger errors or outliers, to verify the feasibility of the new method.

The observation environment is shown in Fig. 1. The baffle plate, located in the south of the rover antenna, is used to reflect the satellite signals. By contrast, the base station is located at the corner of the roof of Wenfa Building in central south university, and its measurement environment is the open-sky.

The number of visible satellites and the values of the corresponding east dilution of precision (EDOP), north dilution of precision (NDOP), and up dilution of precision (UDOP) are shown in Fig. 2. The information about GPS, GLONASS and BDS are plotted by the blue, cyan and red lines, respectively. As shown in Fig. 2, the number of visible satellites is 6–12 for GPS and 7 to 12 for BDS, which is greater than that for GLONASS (5–9). For the combined GPS, GLONASS, and BDS system, the number of visible satellites is 20–30, which is approximately triple that of a single system. Thus, the integrated system theoretically has more redundant observables and better satellite geometry. The mean DOP values for GPS are 0.47 (east), 0.50 (north), and 0.76 (up), whereas those for GLONASS increase to 0.62 (east), 0.51 (north), and 0.98 (up). Although the mean satellite number of GPS and BDS are similar, the mean DOPs of BDS are greater than those of GPS and GLONASS (EDOP: 0.65; NDOP: 0.73; and UDOP: 1.44), which could be attributed to the special constellation of BDS system. These results coincide exactly with those of He's study [12]. Notably, the DOP values are accompanied by severe fluctuation when the number of satellites remains unchanged or is slightly changed. Therefore, the relationship between satellite number and DOP values should be considered.

Compared with the static baseline components, the kinematic positioning errors of different GNSS system with double frequency in the east, north, and up directions are illustrated in Fig. 3. Meanwhile, the corresponding changes of satellite number are also plotted in Fig. 3. To clarify the distinction clearly, the axis scale is quite diverse for different GNSS system.

Fig. 1. Measurement environment (left: base, right: rover).

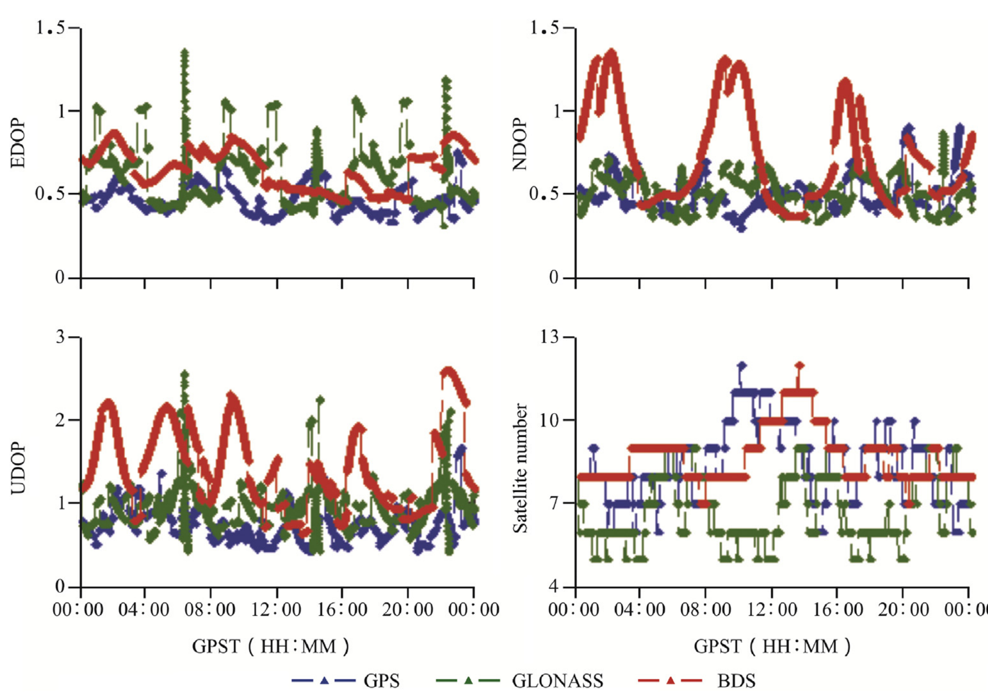

Fig. 2. EDOP, NDOP, UDOP, and the number of available satellites during measurement.

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In the open-sky observation environment, the root mean square (RMS) of kinematic errors is at the same level (Horizontal: better than 3.5 mm; Vertical: better than 10 mm), expect for the GLONASS (East: 60.0 mm; North: 50.9 mm; Up: 282.3 mm). As the observation environment is quite good and many observables are collected in this environment, the positioning accuracy of single GNSS system is not inferior to the integrated system. However, there are significant differences on the positioning accuracy for different GNSS system with the masked environment, especially for the GLONASS system, the precision of which decreases heavily because of the change of the available satellites and the influence of the baffle plate. Compared with the single system, the positioning accuracy of the integrated system remains about the same on account of the redundant observables. As shown in the sub-graph 4, the visible satellite numbers of the single GNSS system, by reason of the masked observation environment, has obvious change, particularly for the BDS system.

Fig. 4 shows the relationship between the RMS values of baseline components and the average condition numbers using different GNSS observables. Evidently, the RMS values are associated with the average condition numbers significantly, especially in the vertical direction. For the single GNSS system, the positioning accuracy of GPS and BDS is approximately the same in the horizontal direction. By contrast, the accuracy of GLONASS is the worst in east, north and up directions. As GPS constellation is distributed evenly in the space, GPS observables have been less affected by the masked environment than any other GNSS system and the RMS value of GPS in up direction is smallest in the single system. For the integrated system, the RMS values are superior to 1 cm and combined GPS/BDS system are better than those of the combined GPS/GLONASS/BDS system due to the fact that GPS and BDS system adopts the CDMA technique, whereas GLONASS adopts the FDMA technique.

Although the condition number can be utilized as an evaluation criterion of observation structure stability and positioning accuracy, we cannot set a hard threshold for the condition number to determine observation structure stability because the positioning accuracy of the GNSS integrated system and single satellite system are at different levels, with the former being better than the latter

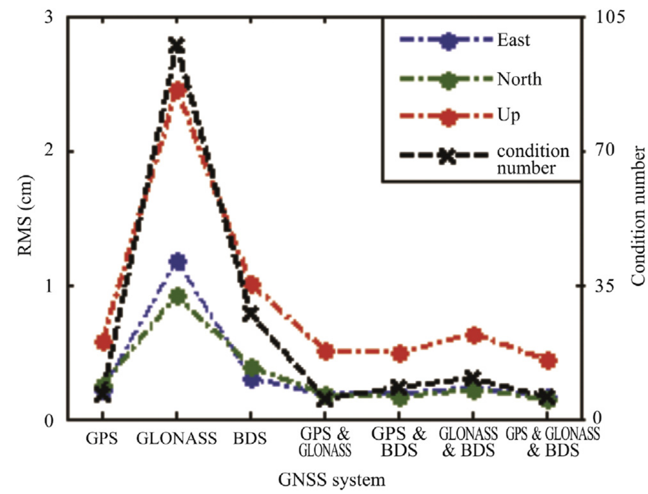

Fig. 4. Relationship between RMS values and condition numbers for different directions.

(see Fig. 5). Thus, a soft threshold is necessary for the condition number to estimate observation structure stability.

Considering that the main difference between the integrated system and single system is the number of visible satellites, this study presents flexible condition number thresholds on the basis of satellite number, as shown in Fig. 5. We use twice the standard deviation (STD) value of the condition numbers as threshold (confidence reaches up to 94.5%), which is plotted with the red dotted line in Fig. 5. Meanwhile, the average condition numbers are used as strong observation structure indicators with the cyan dotted line in Fig. 5. If the condition number of the normal equation is greater than the weak threshold value, then the observation structure is in ill condition, and the positioning solution is unreliable. By contrast, if the condition number is less than the strong threshold value, then the positioning solution is more reliable because the observation structure is strongly stable, i.e., the visible satellites have good distribution.

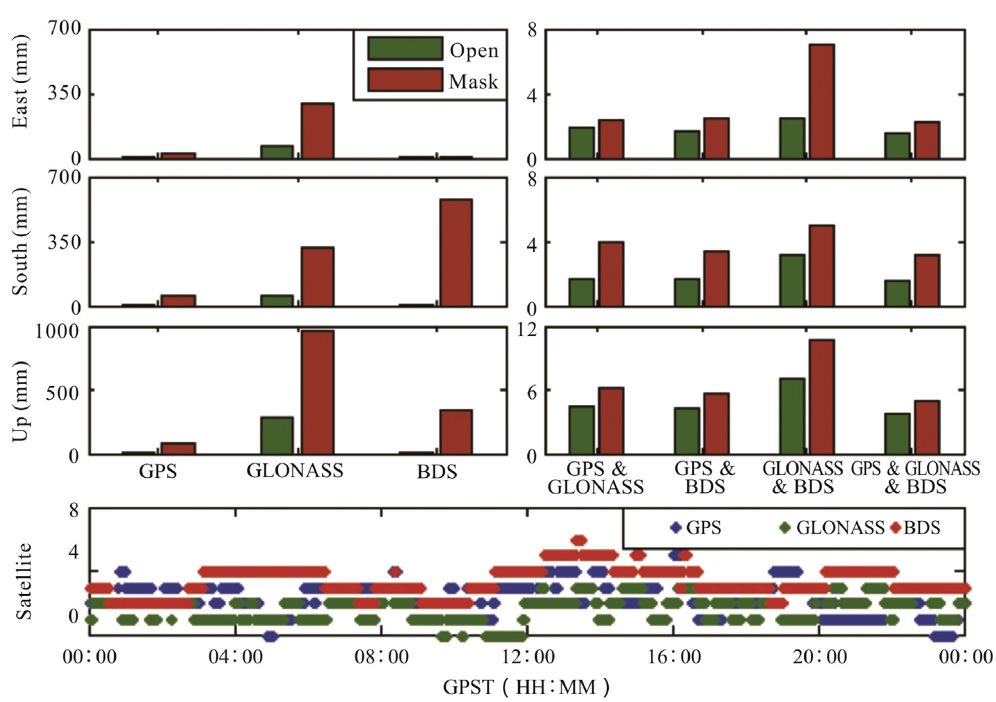

Fig. 3. Comparison of the root mean square of the positioning solutions between open-sky and masked observation environment.

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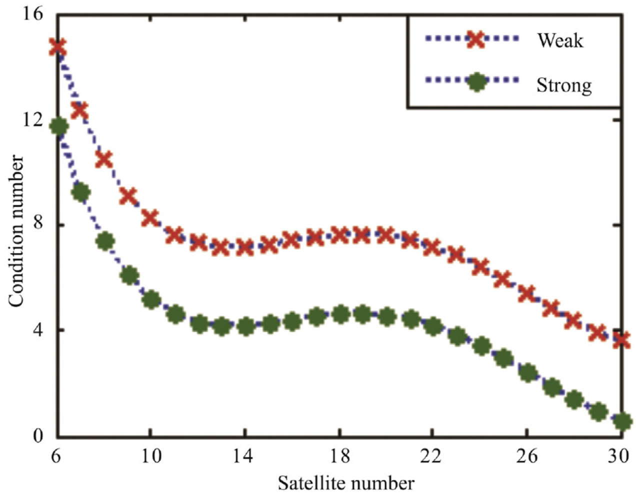

Fig. 5. Fitting figure of condition number thresholds based on satellite number.

6. Conclusion

Considering that the reference satellite significantly affects GNSS relative positioning, we analyzed the advantages and disadvantages of classical reference satellite selection methods. Although satellite elevation and PDOP value can be used to select the reference satellite and serve important function in absolute positioning, these methods cannot reflect the full picture of the satellite pairs and observation structure. The condition number is a better evaluation criterion of the observation structure as opposed to the previous methods. However, condition number cannot reduce the impact of measurement outliers. Therefore, in this study, we present a new method that considers condition number and robust estimation to select the reference satellite and achieve higher positioning accuracy and stable solutions.

The new method is verified by real data, which are collected in masked observation environment. By analyzing kinematic positioning solutions, we reached the conclusion that the new method is more effective for the integrated system than the single system because the former has a large amount of redundant observables, which are conducive to detecting the measurement outliers and ensuring the reliability of the positioning solutions. By comparing the solutions of different satellite systems, GPS and BDS have the same accuracy level, whereas GLONASS has the lowest accuracy level in the horizontal direction. In the vertical direction, GPS accuracy is the highest, whereas GLONASS accuracy is lowest. Based on the statistics of kinematic solutions, the RMS ratio of GPS, GLONASS, and BDS is determined as 1:4:1.5.

In this study, we investigated the reference satellite selection methods in relative positioning and used the measurement data to verify the feasibility of the new method. The proposed method has significant effect on the integrated system because of abundant redundant observables. Thus, this new method can improve the positioning accuracy and reliability of the integrated system which has sufficient visible satellites. We also expect that the proposed method can serve a significant function in the integrated system with a fully equipped Galileo system.

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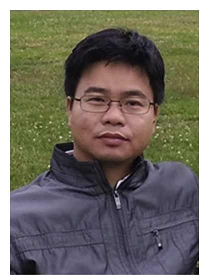

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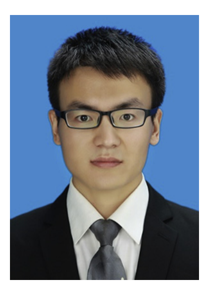

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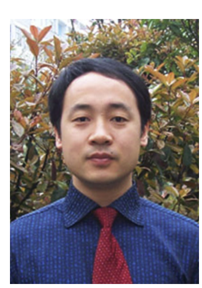

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